

2.1

Proportions

In this lesson you will

- learn several ways for writing a **ratio**
- learn methods for **solving proportions**
- solve problems by writing and solving proportions

The statement “Jackie scored 24 of the team’s 64 points” compares two numbers. The **ratio** of the points Jackie scored to the points the team scored is 24 to 64. You can write the ratio as $24 : 64$ or as a fraction or decimal. The fraction bar means division, so these expressions are equivalent.

$$\frac{24}{64} \quad 24 \div 64 \quad 0.375 \quad \frac{3}{8}$$

Read Example A and the text that follows starting on page 93 of your book. Make sure you understand the difference between a **terminating decimal** and a **repeating decimal**.

A **proportion** is an equation stating that two ratios are equal. Here are some true proportions using the integers 3, 5, 9, and 15.

$$\frac{9}{15} = \frac{3}{5} \quad \frac{15}{9} = \frac{5}{3} \quad \frac{5}{15} = \frac{3}{9} \quad \frac{15}{5} = \frac{9}{3}$$

You can check that the proportions are true by finding the decimal equivalent of each side. The proportion $\frac{3}{15} = \frac{5}{9}$ is not true; 0.2 is not equal to $0.\bar{5}$.

In algebra, a **variable** stands for an unknown number or numbers. In the proportion $\frac{R}{16} = \frac{1}{4}$, you can replace the variable R with any number, but only one number, 4, will make the proportion true.

Investigation: Multiply and Conquer

Steps 1–4 When you multiply both sides of an equation by the *same number*, the sides remain equal. You can use this idea to solve proportions with a variable in one of the numerators. For example, you can solve $\frac{M}{19} = \frac{56}{133}$ by multiplying both sides by 19.

$$\frac{M}{19} = \frac{56}{133}$$

$$19 \cdot \frac{M}{19} = \frac{56}{133} \cdot 19 \quad \text{Multiply both sides by 19.}$$

$$M = \frac{56}{133} \cdot 19 \quad \frac{19}{133} \text{ is equivalent to } \frac{1}{7}.$$

$$M = 8 \quad \text{Multiply and divide.}$$

You can check that the solution is correct by replacing M with 8 and making sure the resulting proportion, $\frac{8}{19} = \frac{56}{133}$, is true.

(continued)

Lesson 2.1 • Proportions (continued)

Here is the solution to part a in Step 2. Try solving parts b–d on your own.

$$\frac{21}{35} = \frac{Q}{20}$$

$$20 \cdot \frac{21}{35} = \frac{Q}{20} \cdot 20 \quad \text{Multiply both sides by 20.}$$

$$20 \cdot \frac{21}{35} = Q \quad \frac{20}{20} \text{ is equivalent to 1.}$$

$$12 = Q \quad \text{Multiply and divide.}$$

Steps 5–7 In Step 5, the ratios in the proportions from Step 2 have been *inverted*. These new proportions have the same solutions as the original proportions. For example, 12 is a solution to both $\frac{21}{35} = \frac{Q}{20}$ and $\frac{35}{21} = \frac{20}{Q}$. (Check that this is true.) You can use this idea to solve proportions with the variable in a denominator. For example, to solve $\frac{20}{135} = \frac{12}{k}$, just invert the ratios to get $\frac{135}{20} = \frac{k}{12}$, and multiply both sides by 12.

Now, read the question and sample solutions in Step 7 and make sure you understand them.

When a problem involves a ratio or percent, you can sometimes solve it by setting up and solving a proportion. Examples B and C in your book present some sample problems. Here is another example.

EXAMPLE | Raj answered 75% of the questions on the algebra midterm correctly. If he got 27 correct answers, how many questions were on the test?

► **Solution** | Let q represent the number of questions on the test. Use the fact that 75% is 75 out of 100 to help you write a proportion. The ratio 27 out of q equals 75 out of 100.

$$\frac{27}{q} = \frac{75}{100} \quad \text{Write the proportion.}$$

$$\frac{q}{27} = \frac{100}{75} \quad \text{Invert both sides.}$$

$$27 \cdot \frac{q}{27} = \frac{100}{75} \cdot 27 \quad \text{Multiply both sides by 27.}$$

$$q = \frac{100}{75} \cdot 27 \quad \frac{27}{27} \text{ is equivalent to 1.}$$

$$q = 36 \quad \text{Multiply and divide.}$$

There were 36 questions on the test.

Capture-Recapture

In this lesson you will

- simulate the **capture-recapture method** for estimating animal populations
- write and **solve proportions**
- solve three types of **percent problems**: finding an unknown percent, finding an unknown total, and finding an unknown part

Wildlife biologists use a method called “capture-recapture” to estimate animal populations. This method involves tagging some animals and then releasing them to mingle with the larger population. Later, a sample is taken. Using the ratio of tagged animals in the sample to total animals in the sample, biologists can estimate the animal population.

Investigation: Fish in the Lake

In this investigation a bag of white beans represents a population of fish in a lake. To simulate the capture-recapture method, you reach into the “lake” and remove a handful of “fish.” You count the fish in the sample. Instead of putting them back, you replace these fish (white beans) with an equal number of “tagged fish” (red beans).

You then allow the fish to mingle (seal the bag and shake it to mix the beans) and then remove another sample. You count all the fish in the sample and the tagged fish in the sample before you return the fish to the lake. By taking several more samples, you can get a good idea of the ratio of tagged fish to total fish in the lake.

One group of students tagged and returned 84 fish. They then took five samples. Here are their results.

Sample number	Number of tagged fish	Total number of fish	Ratio of tagged fish to total fish
1	8	48	$\frac{8}{48} \approx 0.17$
2	24	102	$\frac{24}{102} \approx 0.24$
3	16	86	$\frac{16}{86} \approx 0.19$
4	17	67	$\frac{17}{67} \approx 0.25$
5	16	75	$\frac{16}{75} \approx 0.21$

To estimate the population of fish in this group’s lake, you need to choose one ratio to represent all the samples. You might calculate the median or the mean or use some other method of choosing a representative ratio. For this example, we will use the median of the ratios, which is $\frac{16}{75}$.

(continued)

Lesson 2.2 • Capture-Recapture (continued)

If the fish were mixed well, the fraction of the tagged fish in a sample should be close to the fraction of the tagged fish in the entire population. In other words, the following should be true:

$$\frac{\text{tagged fish in sample}}{\text{total fish in sample}} \approx \frac{\text{tagged fish in population}}{\text{total fish in population}}$$

In this case, there were 16 tagged fish in the sample, 75 total fish in the sample, and 84 tagged fish in the population. So you can estimate the fish population, f , by solving this proportion:

$$\frac{16}{75} = \frac{84}{f}$$

To solve the proportion, invert the ratios and multiply both sides by 84.

$$\frac{75}{16} = \frac{f}{84} \quad \text{Invert both ratios.}$$

$$84 \cdot \frac{75}{16} = \frac{f}{72} \cdot 84 \quad \text{Multiply both sides by 84.}$$

$$393.75 = f \quad \text{Multiply and divide.}$$

So there are about 400 fish in the lake (that is, about 400 beans in the bag).

You can describe the results of capture-recapture situations using percents. The examples in your book show three different kinds of percent problems: finding an unknown percent, finding an unknown total, and finding an unknown part. Be sure to read each example and make sure you understand it. Here is one more example.

EXAMPLE In a lake with 350 tagged fish, the recapture results show that 16% of the fish are tagged. About how many fish are in the lake?

► **Solution** In this case, the variable is the total number of fish in the lake, f . Because 16% of the fish are tagged, there are 16 tagged fish to every 100 fish. You can write this as the ratio $\frac{16}{100}$. The ratio of the total number of tagged fish, 350, to the total number of fish in the lake, f , is about $\frac{16}{100}$.

$$\frac{16}{100} = \frac{350}{f} \quad \text{Write the proportion.}$$

$$\frac{100}{16} = \frac{f}{350} \quad \text{Invert both ratios.}$$

$$350 \cdot \frac{100}{16} = \frac{f}{350} \cdot 350 \quad \text{Multiply both sides by 350.}$$

$$2187.5 = f \quad \text{Multiply and divide.}$$

There are about 2200 fish in the lake.

Proportions and Measurement Systems

In this lesson you will

- find a **conversion factor** to change measurements from centimeters to inches
- use **dimensional analysis** to do conversions involving several steps

If you travel outside the United States, it is helpful to be familiar with the *Système Internationale*, or SI, known in the United States as the metric system. To change from one system of measurement to another, you can use ratios called **conversion factors**.

Investigation: Converting Centimeters to Inches

To find a ratio you can use to convert centimeters to inches and inches to centimeters, first carefully measure the length or width of several different objects in both units. Here are some sample data. You may want to collect your own data or just add a few measurements to this table.

Object	Measurement in inches	Measurement in centimeters
pen	$5\frac{3}{4} = 5.75$	14.7
calculator	3.0	7.6
paper	$8\frac{1}{2} = 8.5$	21.6
paper clip	$1\frac{7}{8} = 1.875$	4.7
pencil	$6\frac{13}{16} = 6.8125$	17.4
desk	30.0	76.2

Enter the measurements in inches into list L1 of your calculator and the measurements in centimeters into list L2. Enter the ratio of centimeters to inches, $\frac{L2}{L1}$, into list L3, and let your calculator fill in the ratio values. (See **Calculator Note II**.) Here is the table for the data above.

L1	L2	L3	3
5.75	14.7	2.556521739	
3	7.6	2.533333333	
8.5	21.6	2.541176471	
1.875	4.7	2.506717033	
6.8125	17.4	2.554117647	
30	76.2	2.54	

L3(1)=2.556521739...			

To find a single value to represent the ratio of centimeters to inches, you can use the median or mean of the ratios in list L3. In this case, both the mean and median are about 2.54. So the ratio of centimeters to inches is $\frac{2.54}{1}$ or 2.54. This number is the conversion factor between inches and centimeters. It means that 1 inch is equivalent to about 2.54 centimeters.

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Lesson 2.3 • Proportions and Measurement Systems (continued)

Using this ratio, you can write and solve a proportion to convert a centimeter measurement to an inch measurement or vice versa. When you set up a proportion, make sure both sides show centimeters to inches or both sides show inches to centimeters.

Here's how to convert
215 centimeters to inches:

$$\begin{aligned}\frac{2.54}{1} &= \frac{215}{x} \\ \frac{1}{2.54} &= \frac{x}{215} \\ 215 \cdot \frac{1}{2.54} &= \frac{x}{215} \cdot 215 \\ 84.6 &\approx x \\ 215 \text{ centimeters is about} \\ 84.6 \text{ inches.}\end{aligned}$$

Here's how to convert
80 inches to centimeters:

$$\begin{aligned}\frac{2.54}{1} &= \frac{x}{80} \\ 80 \cdot \frac{2.54}{1} &= \frac{x}{80} \cdot 80 \\ 203.2 &= x \\ 80 \text{ inches is about} \\ 203.2 \text{ centimeters.}\end{aligned}$$

Some conversions require several steps. The example in your book shows how to use a strategy called **dimensional analysis** to convert a measurement from feet per second to miles per hour. Here is another example that uses dimensional analysis.

EXAMPLE

A car traveled 500 kilometers on 45 liters of gas. Using the facts that 1 mile equals 1.61 kilometers and 1 gallon equals 3.79 liters, express the car's gas consumption in miles per gallon.

► Solution

You can use the given information to express the car's gas consumption as the ratio $\frac{500 \text{ kilometers}}{45 \text{ liters}}$. Using the other facts in the problem, you can create fractions with a value of 1, for example, $\frac{1 \text{ mile}}{1.61 \text{ kilometers}}$. By multiplying the original ratio by such fractions, you can convert the gas-consumption ratio to miles per gallon.

$$\begin{aligned}\frac{500 \text{ kilometers}}{45 \text{ liters}} \cdot \frac{3.79 \text{ liters}}{1 \text{ gallon}} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometers}} &= \frac{1895 \text{ miles}}{72.45 \text{ gallons}} \\ &\approx \frac{26 \text{ miles}}{1 \text{ gallon}} \text{ or } 26 \text{ miles per gallon}\end{aligned}$$

Notice that the fractions equivalent to 1 were chosen so that when the units cancel, the result has miles in the numerator and gallons in the denominator.

2.4

Increasing and Decreasing

In this lesson you will

- use proportions to solve problems involving **percent increase** or **percent decrease**
- explore how increasing or decreasing a rectangle's dimensions by a percent affects diagonal length, perimeter, and area

Many everyday situations—including finding sale prices, taxes, and tips—involve working with percent increases or percent decreases. In this lesson you learn how to use proportions to solve such problems.

EXAMPLE

To celebrate its 21st anniversary, Elm Street Electronics is offering 21% off all merchandise. Conor wants to buy a pair of speakers originally marked \$189. What is the sale price for the speakers?

► Solution

If Conor saves 21%, then he pays $100 - 21$ or 79% of the original price. Because 79% is the same as 79 out of 100, you can find the sale price, S , by using this proportion.

$$\frac{S}{189} = \frac{79}{100}$$

Sale price
Part of original price Conor pays
Original price
The whole original price

To solve the proportion, multiply both sides by 189.

$$189 \cdot \frac{S}{189} = \frac{79}{100} \cdot 189 \quad \text{Multiply both sides by 189.}$$

$$S = \frac{79}{100} \cdot 189 \quad \frac{189}{189} \text{ is equivalent to 1.}$$

$$S = 149.31 \quad \text{Multiply and divide.}$$

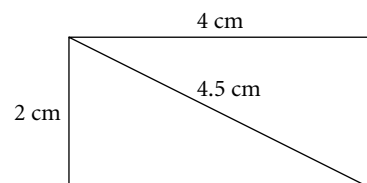
The sale price is \$149.31.

Percent-increase calculations use the same reasoning as percent-decrease calculations. For example, if you add a 15% tip to a dinner bill of \$22, you pay 115% of \$22.

Example B in your book shows you how to use a calculator to do percent-increase calculations for several prices at the same time. Read the example and make sure you understand it.

Investigation: Enlarging and Reducing

Steps 1–4 At right is a rectangle with one of its diagonals drawn.



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Lesson 2.4 • Increasing and Decreasing (continued)

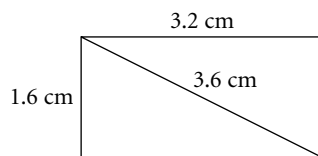
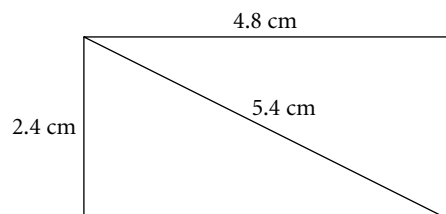
If you increase the length and width by 20%, the new dimensions will be 120% of the original dimensions. You can find the length and width of the enlarged rectangle by solving these proportions.

$$\frac{l}{4} = \frac{120}{100} \quad \text{and} \quad \frac{w}{2} = \frac{120}{100}$$

If you decrease the length and width of the original rectangle by 20%, the new dimensions will be 80% of the original dimensions. You can find the length and width of the reduced rectangle by solving these proportions.

$$\frac{l}{4} = \frac{80}{100} \quad \text{and} \quad \frac{w}{2} = \frac{80}{100}$$

Here are the enlarged and reduced rectangles.



Step 5 This table shows the length, width, diagonal length, perimeter, and area of the rectangles.

The ratio of the diagonal of the enlarged rectangle to the diagonal of the original is $\frac{5.4}{4.5}$ or 1.2 or 120%. If you calculate the ratio of the perimeters, you will also get 1.2 or 120%. So, when the length and width are increased by 20%, the diagonal and the perimeter are increased by 20%.

Rectangle Measurements

	Original dimensions	Enlarged dimensions	Reduced dimensions
Length	4 cm	4.8 cm	3.2 cm
Width	2 cm	2.4 cm	1.6 cm
Diagonal	4.5 cm	5.4 cm	3.6 cm
Perimeter	12 cm	14.4 cm	9.6 cm
Area	8 sq cm	11.52 sq cm	5.12 sq cm

The ratio of the area of the enlarged triangle to the area of the original is $\frac{11.52}{8}$ or 1.44 or 144%. Notice that $1.44 = 1.2^2$. So when the length and width are multiplied by 1.2, the area is multiplied by 1.2^2 .

If you calculate similar ratios to compare the reduced rectangle to the original, you will find that when the length and width are decreased by 20% (that is, multiplied by 0.8), so are the diagonal and perimeter. The area is multiplied by 0.8^2 or 0.64.

Step 7 Read the problem posed in Step 7. If the length l and width w of a tabletop are both reduced by half, then the new area A is one-fourth the original area. To see why, do the area calculation for the reduced tabletop.

$$A = \frac{1}{2}l \cdot \frac{1}{2}w = \frac{1}{4}lw$$

So, if 100 dominoes cover the original tabletop, 25 will cover the reduced tabletop.

Circle Graphs and Relative Frequency Graphs

In this lesson you will

- create **circle graphs**
- compute **relative frequencies**
- create **relative frequency bar graphs** and **relative frequency circle graphs**

Both bar graphs and circle graphs summarize data that is grouped into categories.

Relative frequency graphs show the percent of the total value each category represents. In this lesson you will see how to make relative frequency circle graphs and bar graphs.

Investigation: Circle Graphs and Bar Graphs

The bar graph on page 116 of your book shows the land area of each of the seven continents. You can use the graph to approximate the area of each continent and the total area.

To convert the data into a circle graph, you need to figure out the angle measure for each section of the graph. To do this, use the fact that there are 360 degrees in a circle.

For example, to find the number of degrees in the section representing Australia, solve this proportion.

$$\frac{\text{Land area of Australia}}{\text{Total land area of all continents}} = \frac{\text{Angle measure for Australia section}}{\text{Total of angle measures for all sections}}$$

$$\frac{7}{147} = \frac{x}{360}$$

Continent	Land area (millions of km ²)
Australia	7
Europe	9
Antarctica	14
South America	18
North America	24
Africa	30
Asia	45
Total	147

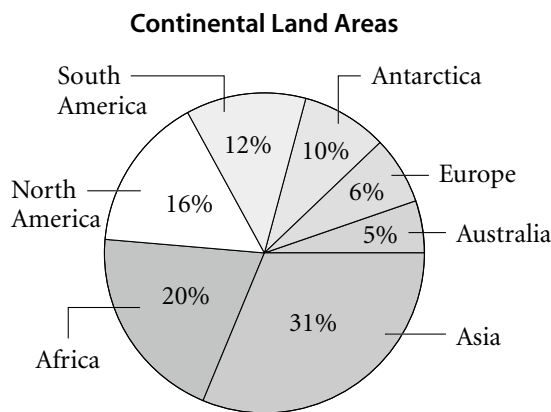
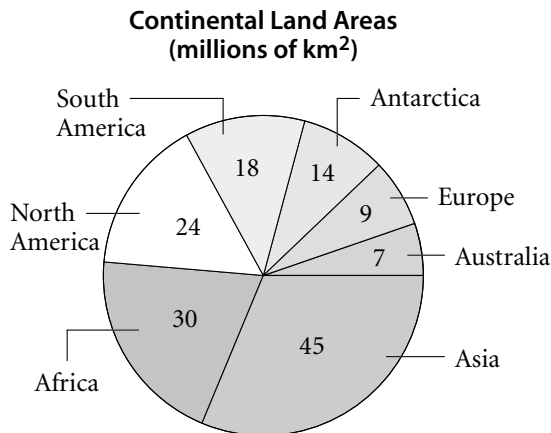
The following table shows the angle measure for each section:

Continent	Angle measure
Australia	17°
Europe	22°
Antarctica	34°
South America	44°
North America	59°
Africa	73°
Asia	110°

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Lesson 2.5 • Circle Graphs and Relative Frequency Graphs (continued)

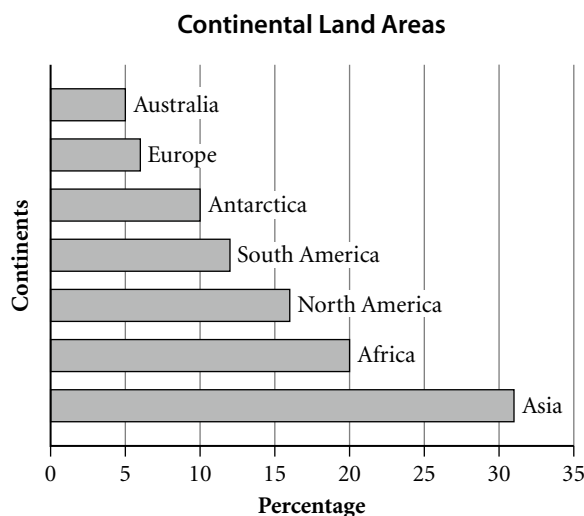
On the left below is the finished graph. To change the graph to a relative frequency circle graph, label each section with the percent of the total area the continent makes up. You can calculate the percents by writing and solving proportions. For example, to find the percent of the total area Australia makes up, you could solve the proportion $\frac{7}{147} = \frac{a}{100}$. On the right is the same circle graph with the sections labeled with percents.



In both circle graphs, the relative size of each section indicates the portion of the total area that continent makes up.

A relative frequency bar graph for this data shows percents of the total land area rather than the land area itself. Here is the completed relative frequency bar graph.

Notice that, like box plots, relative frequency plots do not show actual data values. For example, both of these relative frequency graphs show that Asia makes up 31% of the total land area of the continents, but neither graph indicates what the land area of Asia is.



The example in your book walks you through the steps for creating a relative frequency circle graph and bar graph for a different set of data. In the example, a calculator is used to compute the angle measures for the circle graphs and to calculate relative frequencies. Read through this example and follow along on your calculator.

Probability Outcomes and Trials

In this lesson you will

- calculate **observed probabilities** or **relative frequencies** for events
- calculate **theoretical probabilities** for events
- compare relative frequencies to theoretical probabilities

A **probability** of an event is a number between 0 and 1 (or 0% and 100%) that expresses the chance the event will happen. You can find a probability by calculating the ratio of the number of ways the event can occur to the total number of ways under consideration. For example, the probability of tossing a coin and getting heads is $\frac{1}{2}$ because one of the two possible tosses is heads.

EXAMPLE

Joe works in a train station selling coffee and orange juice to commuters. Last Tuesday he sold 60 large coffees, 25 small coffees, 45 large orange juices, and 20 small orange juices. If this distribution accurately reflects the tastes of his customers, what is the probability his first customer next Tuesday will buy a large orange juice? A coffee? Tea?

► Solution

The probability that the customer will buy a large orange juice can be expressed as the ratio

$$\frac{\text{number of customers who bought a large orange juice}}{\text{total number of customers}} = \frac{45}{150} = 0.3$$

The probability that the customer will buy coffee is

$$\frac{\text{number of customers who bought coffee}}{\text{total number of customers}} = \frac{60 + 25}{150} = \frac{85}{150} \approx 0.57$$

None of the customers bought tea because it was not a choice. So the probability that the first customer will buy tea is

$$\frac{\text{number of customers who bought tea}}{\text{total number of customers}} = \frac{0}{150} = 0$$

Now, read the example in your book and the text that follows, which explains the terms **trial**, **outcome**, **observed probability**, and **theoretical probability**. Think about how each of these terms applies to the example above.

Investigation: Candy Colors

This investigation involves finding the probabilities of selecting various colors from a bag of candy. By conducting an experiment, you can find observed probabilities, or *relative frequencies*. By counting the number of candies of each color, you can find theoretical probabilities.

Steps 1–2 To conduct the experiment, you randomly choose a candy from the bag, record the color, and return the candy to the bag. You repeat this process 40 times. The total number of times each color is selected is called its **experimental frequency**.

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Lesson 2.6 • Probability Outcomes and Trials (continued)

From the experimental frequencies and the total number of trials (40), you can calculate the observed probability, or relative frequency, of each color. For instance, the observed probability of choosing a red candy will be

$$\frac{\text{number of red candies drawn}}{\text{total number of trials}}$$

Here are the data and observed probabilities that one group found.

	Experimental Outcomes						Total
	Red	Orange	Brown	Green	Yellow	Blue	
Tally							40
Experimental frequency	3	13	8	5	10	1	40
Observed probability (relative frequency)	$\frac{3}{40} = 7.5\%$	$\frac{13}{40} = 32.5\%$	$\frac{8}{40} = 20\%$	$\frac{5}{40} = 12.5\%$	$\frac{10}{40} = 25\%$	$\frac{1}{40} = 2.5\%$	100%

Because the table accounts for every possible color, the probabilities add to 1, or 100%.

Steps 3–6 The group who collected the data above dumped out all the candies and counted the number of each color. They then calculated the theoretical probability of drawing each color. For example,

$$P(\text{red}) = \frac{\text{number of red candies in the bag}}{\text{total number of candies in the bag}}$$

Here are their results.

	Outcomes						Total
	Red	Orange	Brown	Green	Yellow	Blue	
Number of candies	2	14	14	10	12	4	56
Theoretical probability	$\frac{2}{56} = 3.6\%$	$\frac{14}{56} = 25\%$	$\frac{14}{56} = 25\%$	$\frac{10}{56} = 17.9\%$	$\frac{12}{56} = 21.4\%$	$\frac{4}{56} = 7.1\%$	100%

In this situation, each candy is equally likely to be selected, but some colors have a higher probability of being chosen than others. The data shows that orange and brown are most likely to be chosen and that red is least likely. Notice that the theoretical probabilities, like the relative frequencies, add to 100%.

Compare the theoretical and experimental probabilities. For example, the observed probabilities predict orange will be chosen about 1 out of 3 times. The theoretical probabilities predict orange will be chosen 1 out of 4 times. In general, the more trials you conduct, the closer the observed probabilities will be to the theoretical probabilities.

Random Outcomes

In this lesson you will

- calculate observed probabilities for a **random** process
- use a calculator to **simulate** coin tosses
- make a graph comparing observed probabilities of tossing heads to the theoretical probability

A process is **random** if you can't predict exactly what will happen on the next trial. Read the introductory text and the example in your book. The example shows that sometimes, even when you cannot predict the exact outcome of a situation, you can use collected results to predict what will happen over the long run. Here is another example.

EXAMPLE

One afternoon, Johnna kept track of the number of cars and trucks that passed by her apartment window over an hour period. (She considered minivans and sports-utility vehicles to be trucks.) She counted 72 cars and 40 trucks. Use these results to predict approximately how many of the next 100 vehicles that pass by will be trucks.

► Solution

The observed probability that a passing vehicle will be a truck is

$$\frac{\text{number of trucks}}{\text{total number of vehicles}} = \frac{40}{112} \approx 36\%$$

Johnna can calculate the *probability* that the next vehicle will be a truck, but she can't know for sure. From Johnna's perspective the event is random. Because the observed probability is 36%, Johnna can expect that 36 of the next 100 vehicles that pass by her window will be trucks.

When you toss a coin once, you cannot predict whether it will show heads or tails because the outcome is random. You do know, however, that there are two equally likely outcomes—heads or tails. Therefore, the theoretical probability of getting a head is $\frac{1}{2}$.

Investigation: Calculator Coin Toss

In this investigation, you use your calculator to **simulate** 100 coin tosses. You then create a scatter plot to compare the theoretical probability of getting heads to an observed probability for 100 trials.

Steps 1–4 Read and follow along with Steps 1–4 in your book. These steps guide you to generate 100 coin tosses. When you are finished, your calculator table will show this information.

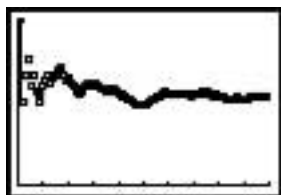
- List L1 will show the trial number.
- List L2 will show the result of each toss, with 0 representing tails and 1 representing heads. For example, the table in your book shows that the result of trial 1 was a tail and the result of trial 2 was a head.

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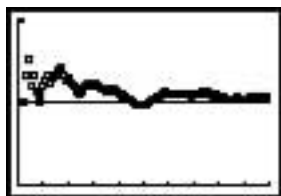
Lesson 2.7 • Random Outcomes (continued)

- List L₃ will show the number of heads tossed so far. In the example in your book, three heads were tossed in the first seven trials.
- List L₄ will show the observed probability, calculated after each trial. In the example in your book, three heads were tossed in the first seven trials, so the observed probability after seven trials is $\frac{3}{7}$ or about 0.43.

Steps 5–8 You can make a scatter plot showing the observed probability after each trial. Use the values in list L₁ (the trial numbers) as the x -values and the values in list L₄ (the observed probabilities) as the y -values. Because there are 100 trials, you will want to set the window to show x -values from 0 to 100. Because the highest possible probability is 1, set the maximum y -value to 1. Your plot might look something like this.



Enter $\frac{1}{2}$, the theoretical probability of tossing a head, into Y₁ on the Y= screen. This graphs the line $y = \frac{1}{2}$ on the same screen as your scatter plot.



By looking at how close the points are to the line, you can compare the observed probabilities to the theoretical probability. Notice that as the number of trials increases, the points get closer and closer to the line; that is, the observed probabilities get closer and closer to the theoretical probability. Add data for 100 more trials to your table and make another scatter plot. You should observe that the points get even closer to the line.

The more times you toss a coin, the closer the observed probability of getting heads will be to $\frac{1}{2}$. However, even though a pattern emerges in the long run, it will not help you predict a particular outcome. When you flip a coin, you know the theoretical probabilities for heads and tails. In some situations, you cannot calculate theoretical probabilities. In such cases, you can perform many trials and determine observed probabilities based on your experimental results.